Minimal Coverability Tree Construction Made Complete and Efficient

Alain Finkel[1,3], Serge Haddad[1,2], and **Igor Khmelnitsky**[1,2]

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[3] IUF, France

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Banana Land

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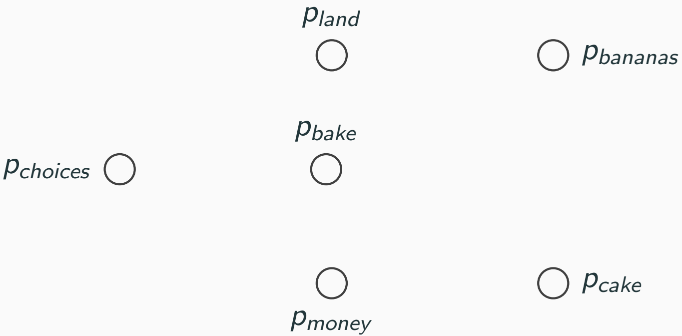

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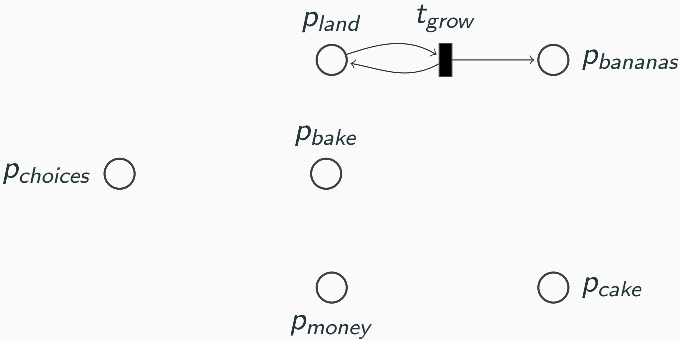

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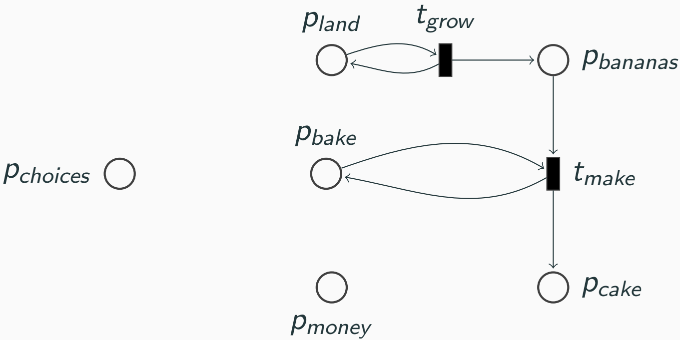

$$\mathbf{m}_0 \xrightarrow{t_1} \mathbf{m}_1 \xrightarrow{t_{grow}} \mathbf{m}_2$$

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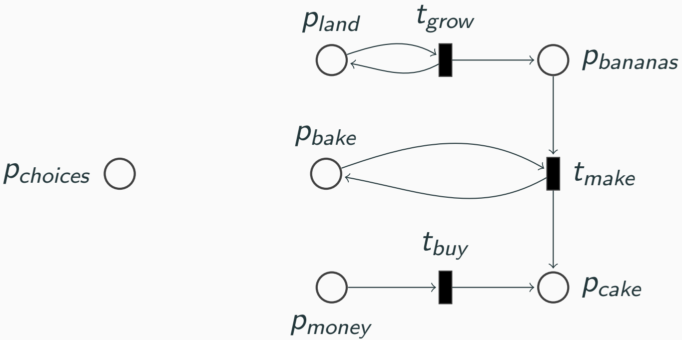

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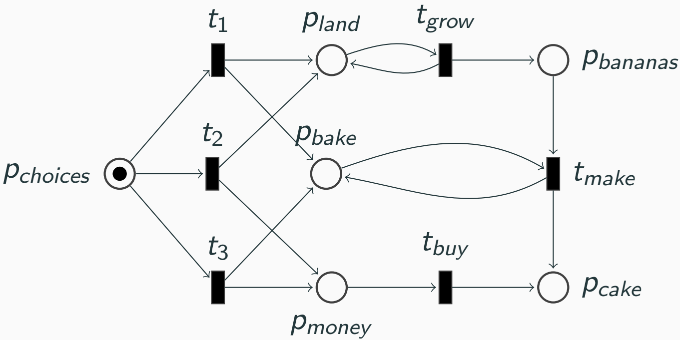

$\mathbf{m}_0$

$\mathbf{m}_0 = p_{choices}$

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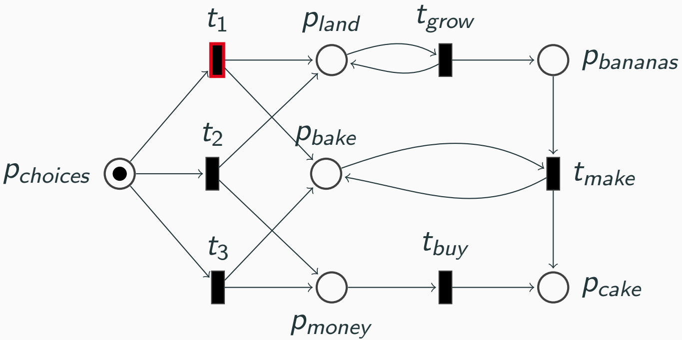

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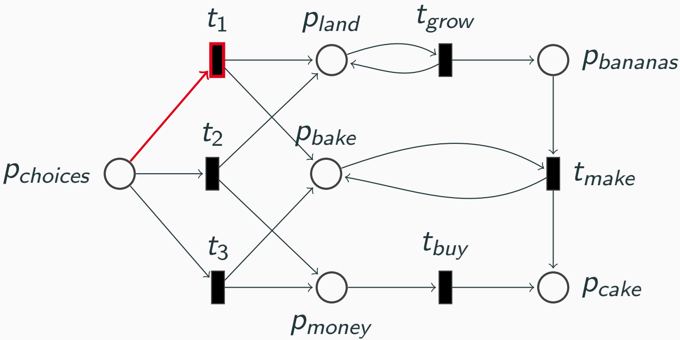

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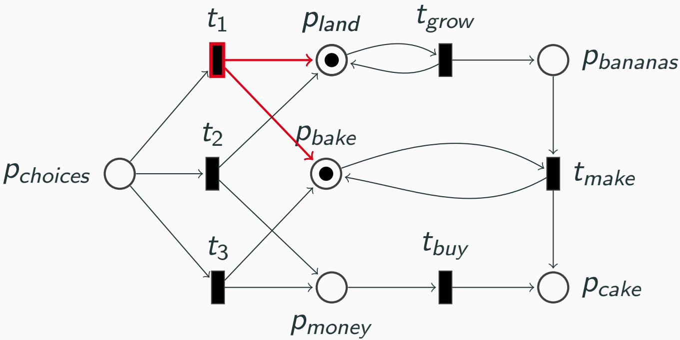

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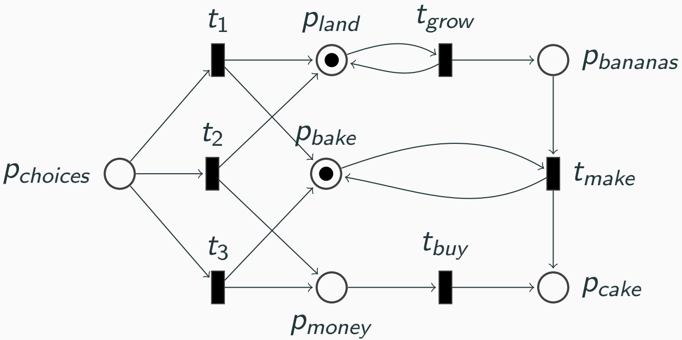

$$\mathbf{m}_0 \xrightarrow{t_1} \mathbf{m}_1$$

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$\mathbf{m}_1 = p_{land} + p_{bake}$

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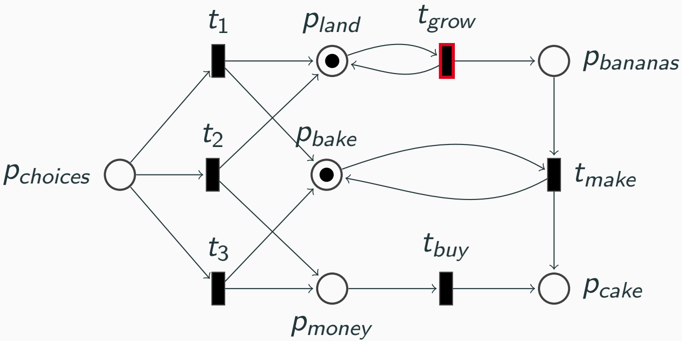

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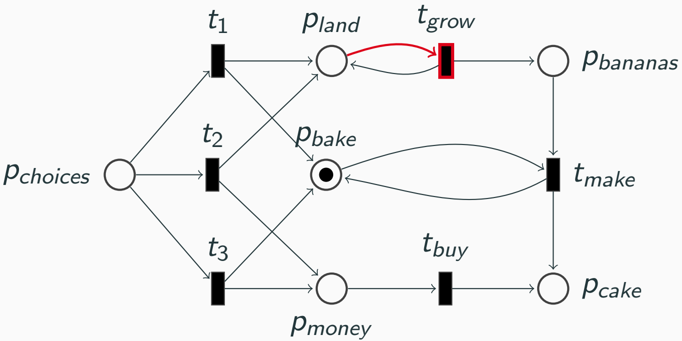

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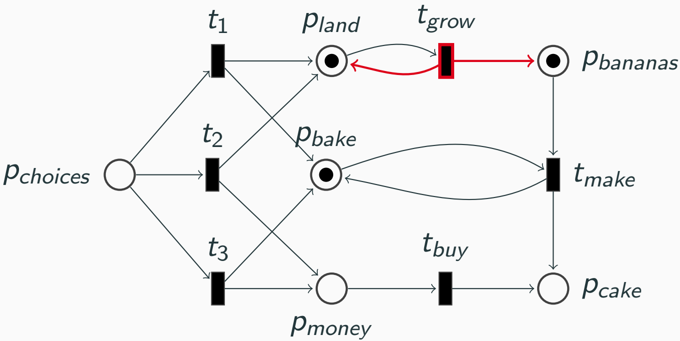

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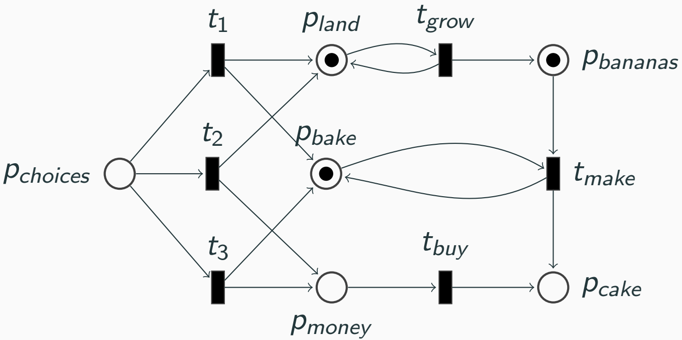

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Petri Nets & Coverability

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Reachability:

Input: $(\mathcal{N}, \mathbf{m}_0, \mathbf{m})$; Output: $\exists ? \mathbf{m}_0 \xrightarrow{\sigma} \mathbf{m}$

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- Safety

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Coverability set: Downward closure of the reachability set

- Parameterized coverability
- Boundedness

Reachability and Coverability in Petri Nets

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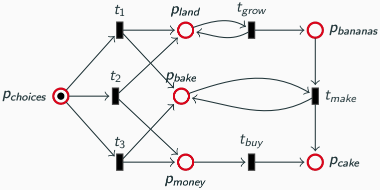
Petri Net:

$$\mathcal{N} = \langle P, T, Pre, C \rangle$$



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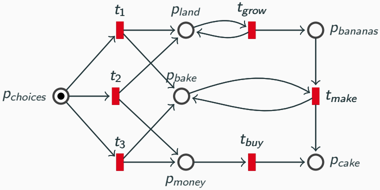

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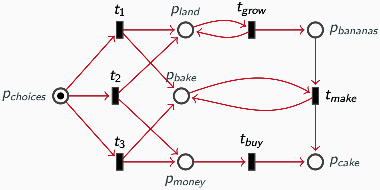

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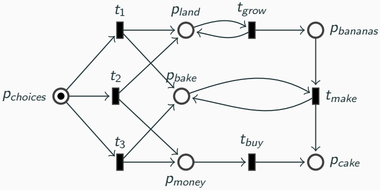

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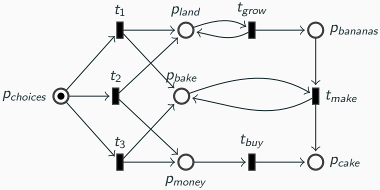

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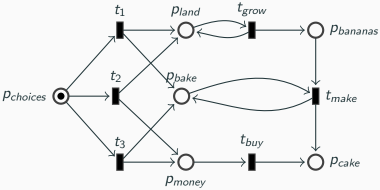

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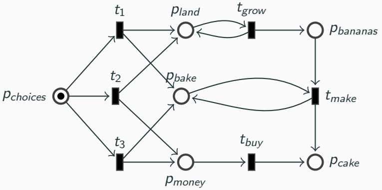

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$$Cover(\mathcal{N}, \mathbf{m}_0) \stackrel{def}{=} \downarrow Reach(\mathcal{N}, \mathbf{m}_0)$$

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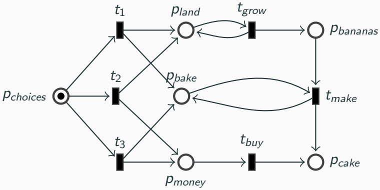

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Questions:

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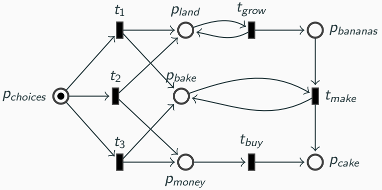

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Questions:

1. Does there exist a finite representation of $Cover(\mathcal{N}, \mathbf{m}_0)$?
2. How to compute it?

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Theorem (Erdős)
There exists a finite (minimal) set of ω-markings $Clover(\mathcal{N}, \mathbf{m}_0)$ s.t.

$$Cover(\mathcal{N}, \mathbf{m}_0) = \bigcup_{\mathbf{m} \in Clover(\mathcal{N}, \mathbf{m}_0)} [\![\mathbf{m}]\!]$$

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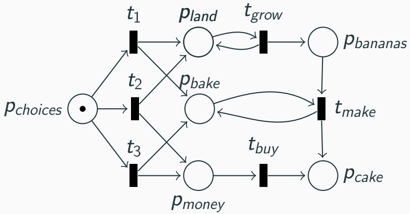
Revisited question: Can one build $Clover(\mathcal{N}, \mathbf{m}_0)$?

First Steps

Reachability Tree



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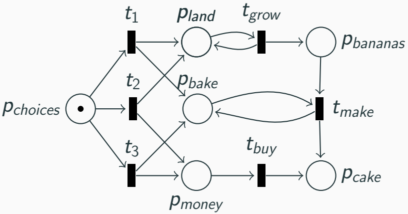

p_C

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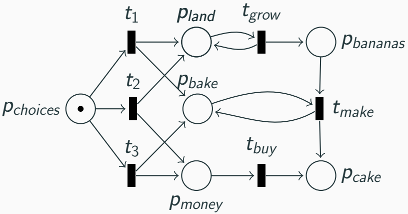

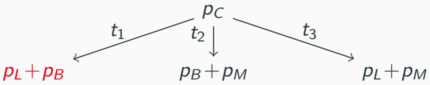

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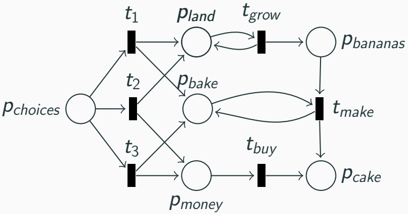

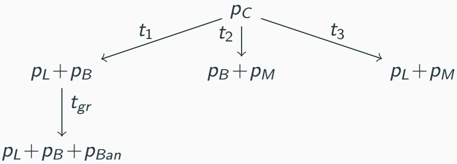

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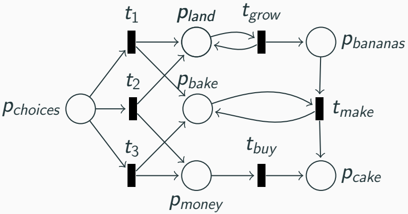

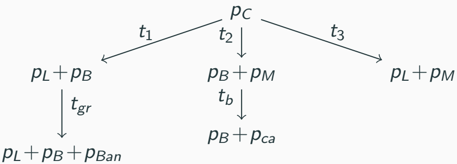

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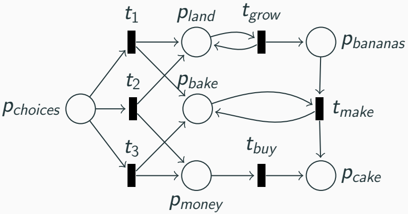

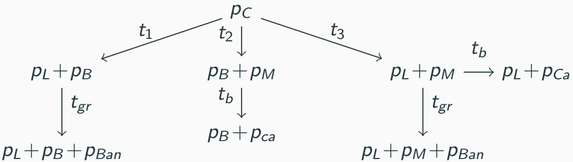

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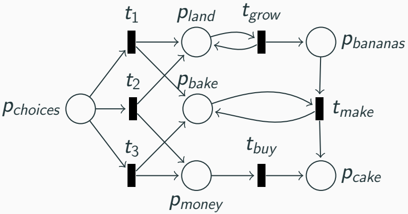

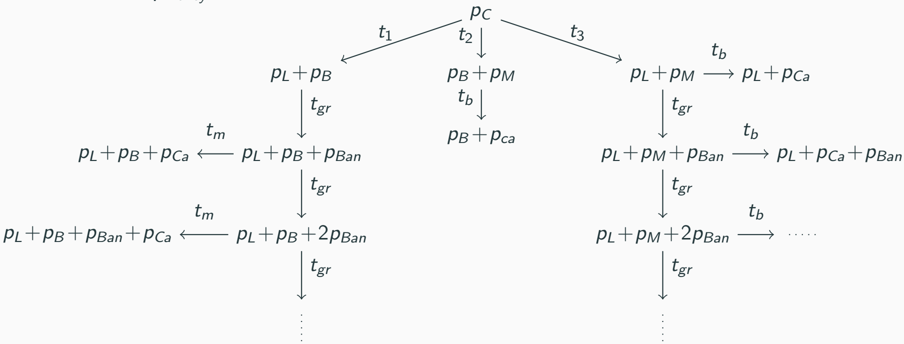

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Reachability enumeration algorithm:

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Explore(v)

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Correctness proof sketch:

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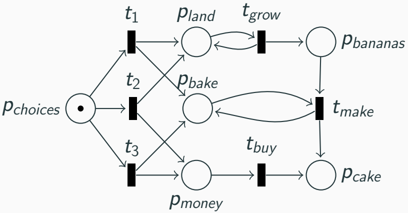
  and applying fairness.

K&M Coverability Tree



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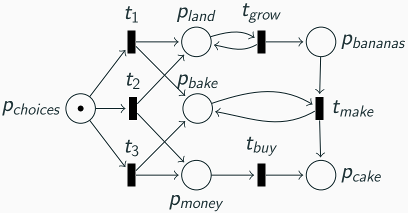

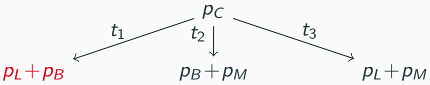

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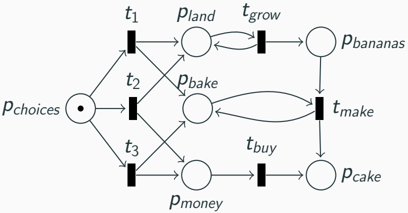

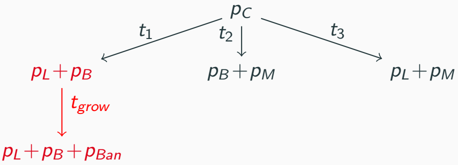

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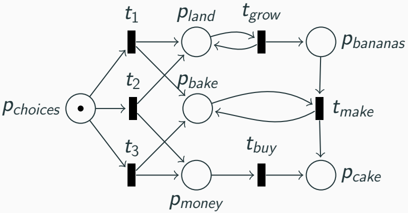

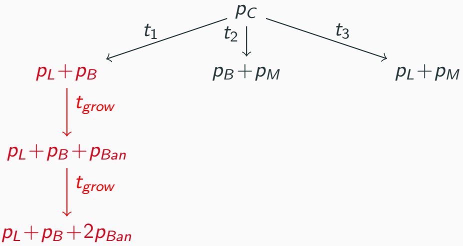

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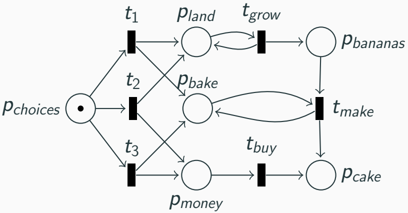

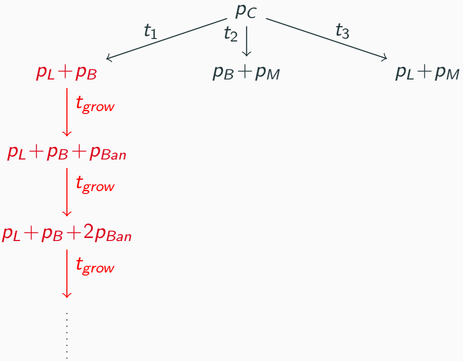

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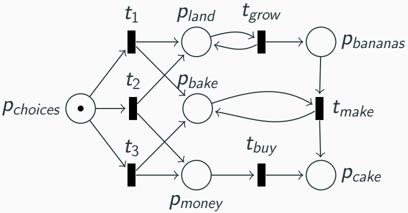

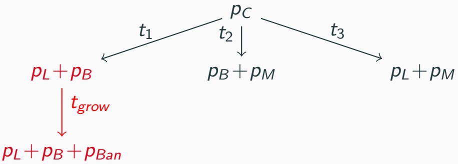

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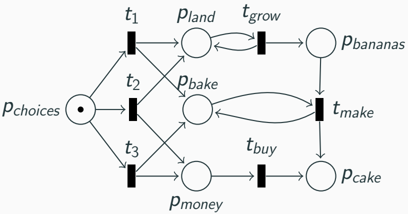

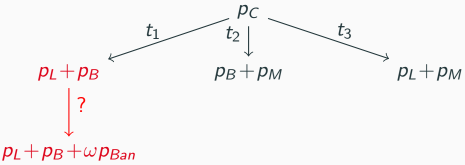

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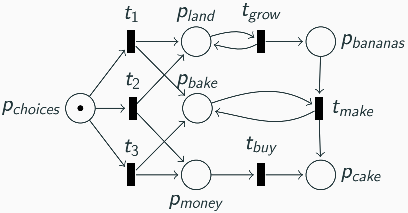

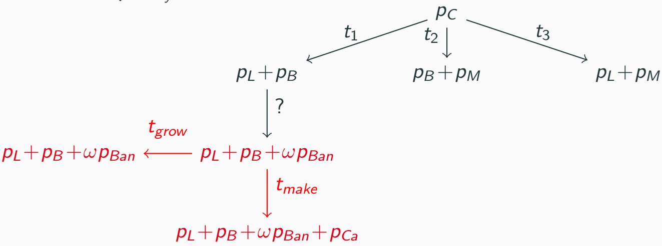

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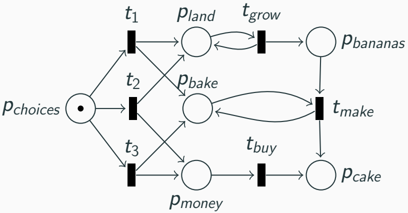

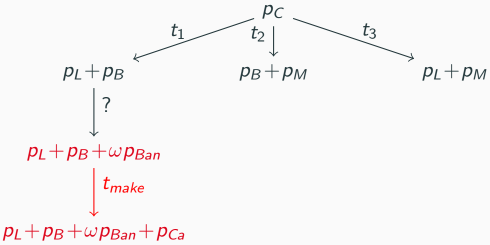

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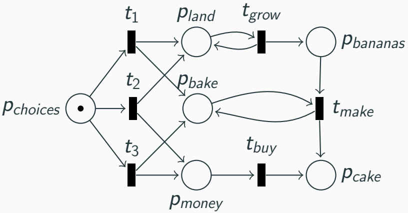

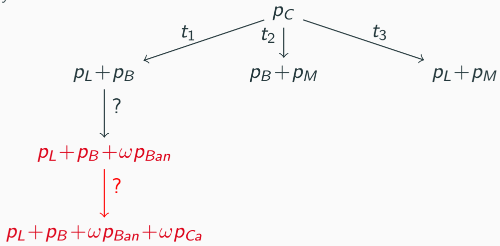

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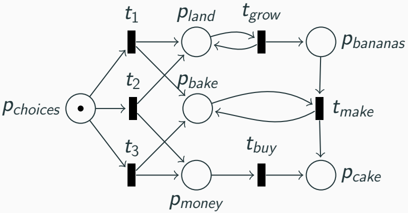

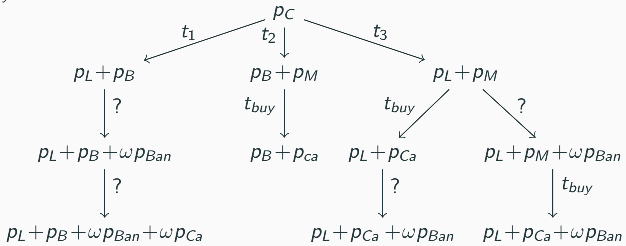

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Correctness proof

- The original proof of K&M-algorithm is incomplete [Hack-74].
- Formal COQ proof of K&M-algorithm [Yamamoto-17].
- Hard to generalize the proof to variants.

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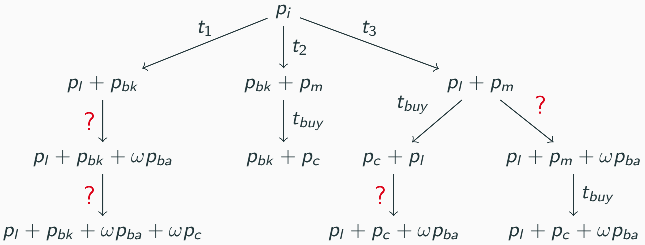

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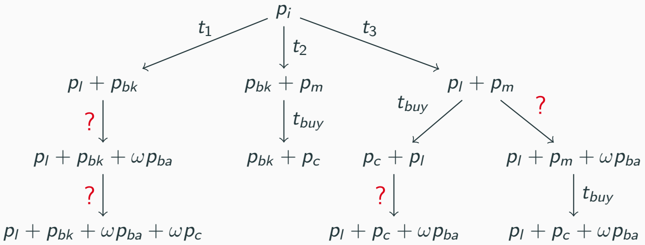

For all edges $u \xrightarrow{?} v$, there does not exist $t \in T$ s.t. $\lambda(u) \xrightarrow{t} \lambda(v)$.

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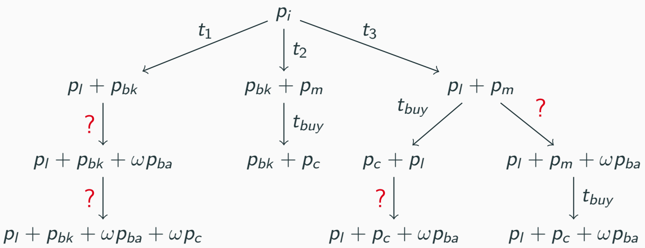

For all edges $u \xrightarrow{?} v$, there does not exist $\sigma \in T^*$ s.t. $\lambda(u) \xrightarrow{\sigma} \lambda(v)$.

Abstractions and Accelerations

$\omega-$Transitions

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Syntax An *ω-transition* **a** is defined by $\mathbf{Pre}(\mathbf{a}) \in \mathbb{N}_\omega^P, \mathbf{C}(\mathbf{a}) \in \mathbb{Z}_\omega^P$ where:

$$\mathbf{Pre}(\mathbf{a}) + \mathbf{C}(\mathbf{a}) \geq 0 \text{ and } \forall p, \text{ s.t. } \mathbf{Pre}(p, \mathbf{a}) = \omega \Rightarrow \mathbf{C}(p, \mathbf{a}) = \omega.$$

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$$\mathbf{m} \xrightarrow{\mathbf{a}} \text{ if } \mathbf{Pre}(\mathbf{a}) \leq \mathbf{m}, \text{ and } \mathbf{m} \xrightarrow{\mathbf{a}} \mathbf{m} + \mathbf{C}(\mathbf{a}).$$

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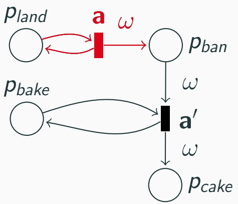
Concatenation (by example)

- $\mathbf{Pre}(\mathbf{a}) = p_{land}, \mathbf{C}(\mathbf{a}) = \omega p_{ban}$



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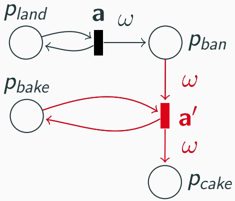

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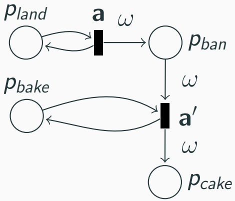

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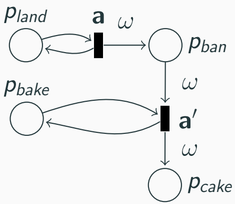

$$\mathbf{m} \xrightarrow{\mathbf{aa}'} \mathbf{m}' \text{ if and only if } \mathbf{m} \xrightarrow{\mathbf{a}\cdot\mathbf{a}'} \mathbf{m}'.$$

Coverability Abstraction

An ω-transition $\mathbf{a}$ is an *abstraction* if for all n, there exists $\sigma_n \in T^*$ s.t. for all p where $\mathbf{Pre}(p, \mathbf{a}) \neq \omega$:

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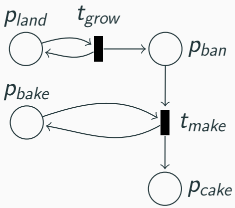

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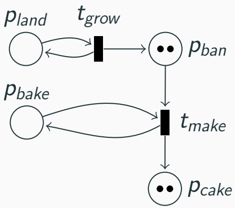

An ω-transition $\mathbf{a}$ is an *abstraction* if for all n, there exists $\sigma_n \in T^*$ s.t. for all p where $\mathbf{Pre}(p, \mathbf{a}) \neq \omega$:

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- If $\mathbf{C}(p, \mathbf{a}) = \omega \Rightarrow \mathbf{C}(p, \sigma_n) \geq n$.

Illustration

- $\mathbf{Pre}(\mathbf{a}) = p_{land} + p_{bake}$,
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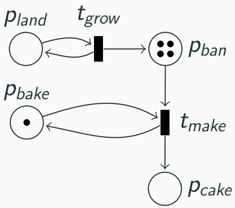
- $\sigma_2 = \quad t_{make}^2$



Coverability Abstraction

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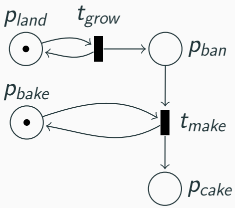

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- $\mathbf{Pre}(\mathbf{a}) = p_{land} + p_{bake}$,
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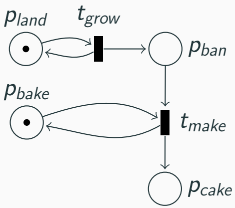
- $\sigma_2 = t_{grow}^4 t_{make}^2$
- $\sigma_n = t_{grow}^{2n} t_{make}^n$
  $\mathbf{Pre}(\sigma_n) = p_{land} + p_{bake}$,
  $\mathbf{C}(\sigma_n) = np_{ban} + np_{cake}$



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Example

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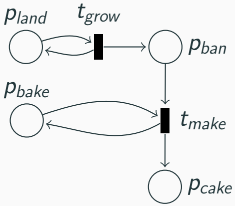

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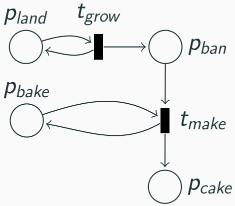

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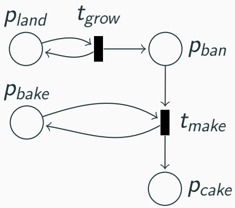

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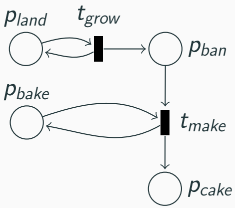

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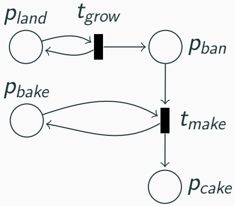
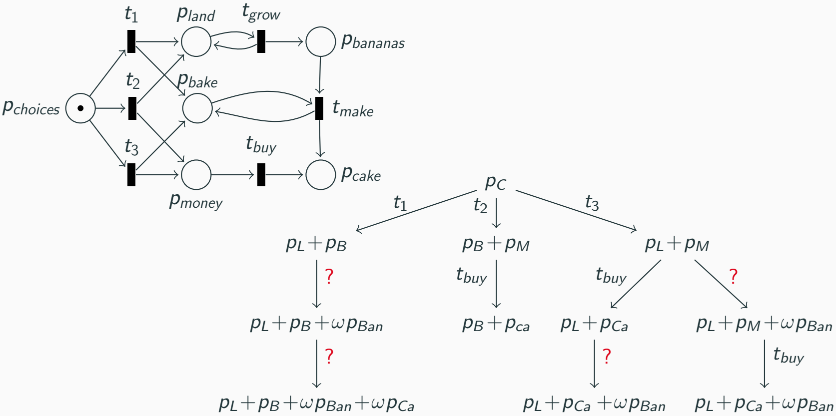


Completing the K&M tree



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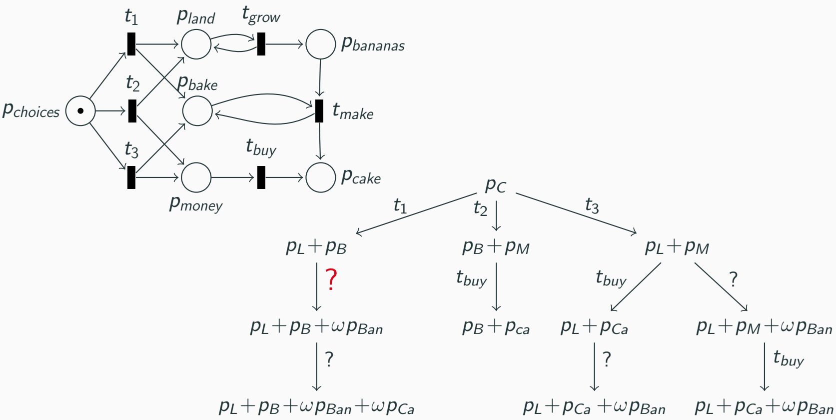

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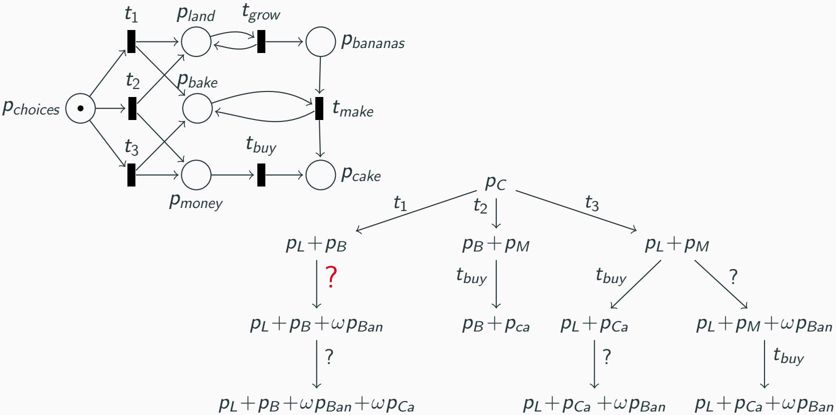

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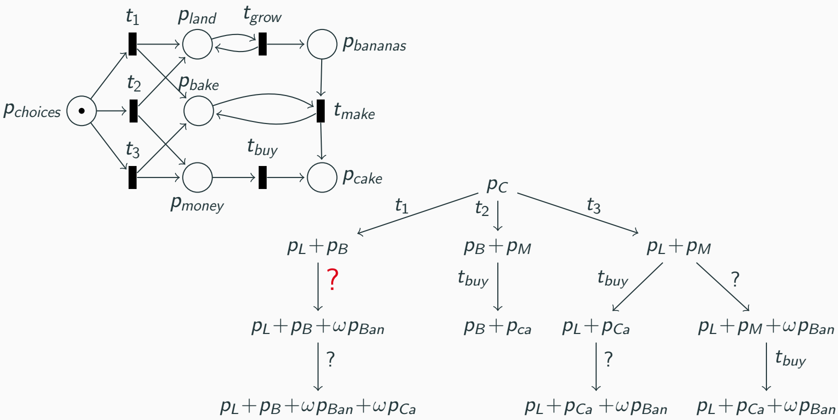

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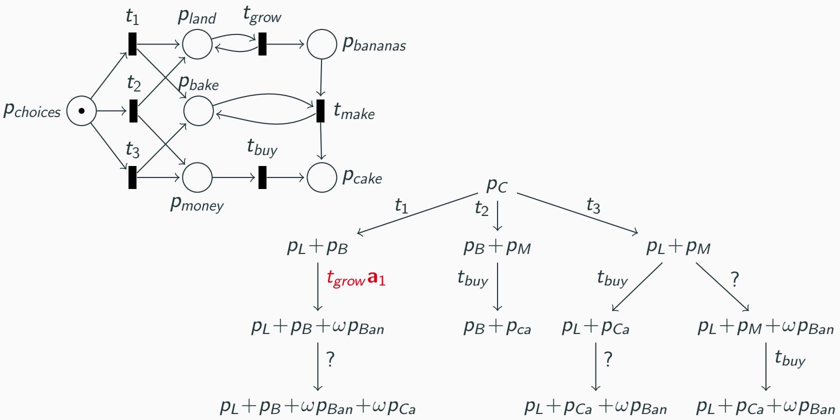

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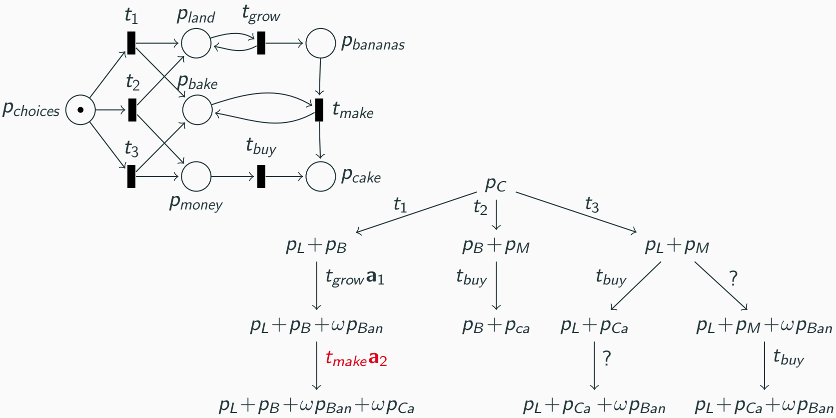

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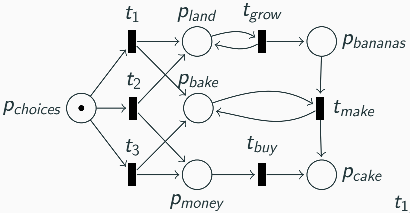

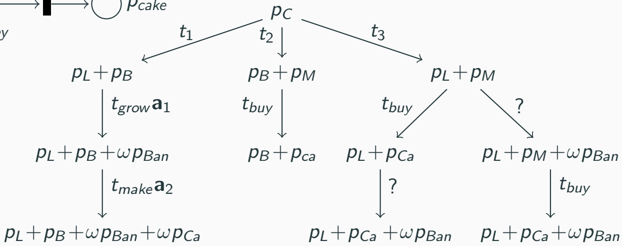

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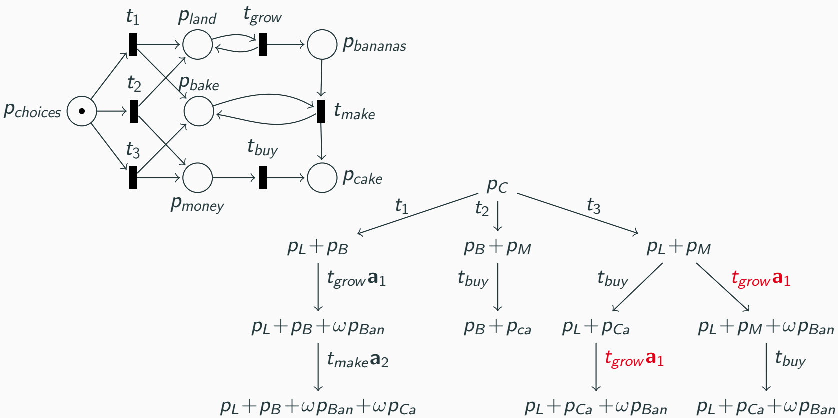

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$\forall \mathbf{m} \in \mathit{Reach}(\mathcal{N}, \mathbf{m}_0)$, $\exists u \in V$ s.t.:

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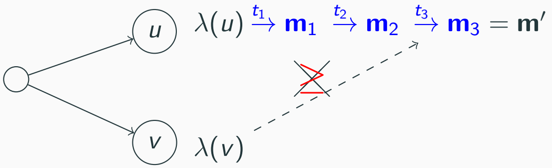
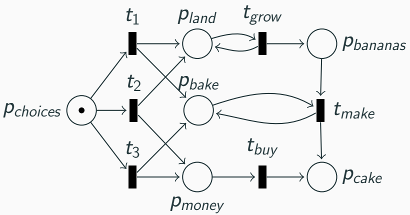
- $u \in$ Front, $\exists \sigma$ and exploring sequence s.t. $\lambda(u) \xrightarrow{\sigma} \mathbf{m}' \geq \mathbf{m}$

Exploring sequence(Illustration):

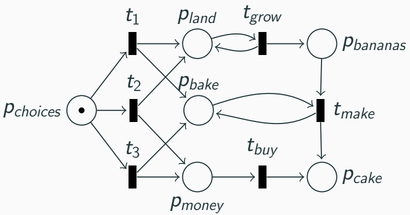


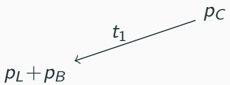
K&M With Accelerations



p_C

K&M With Accelerations





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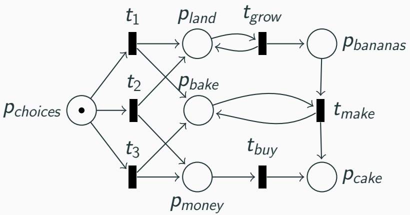
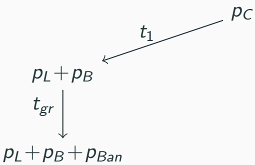
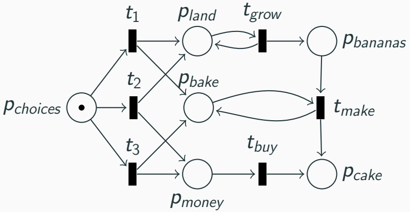




K&M With Accelerations



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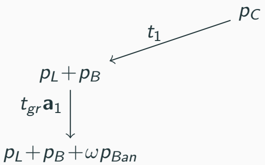

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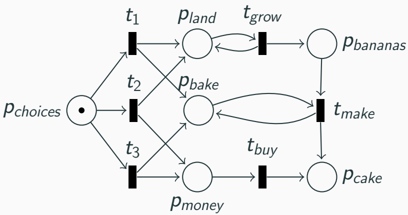

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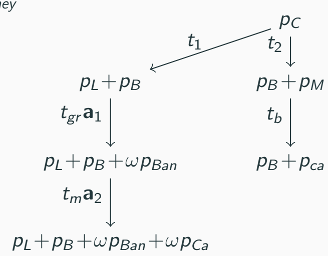

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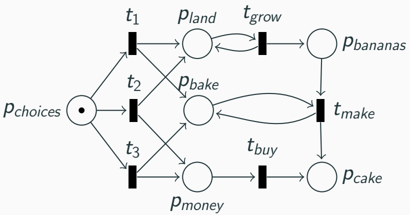

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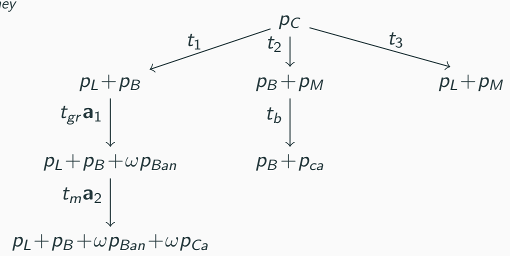

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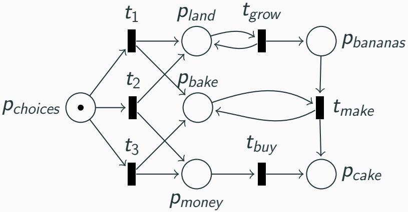

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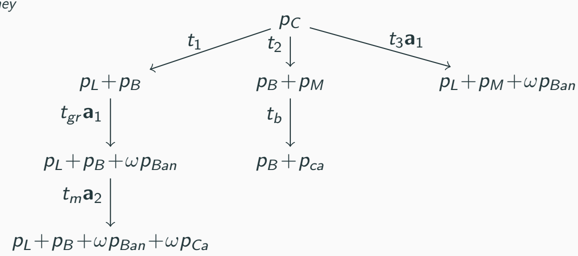

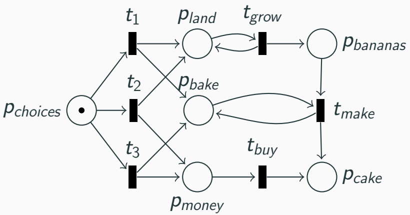

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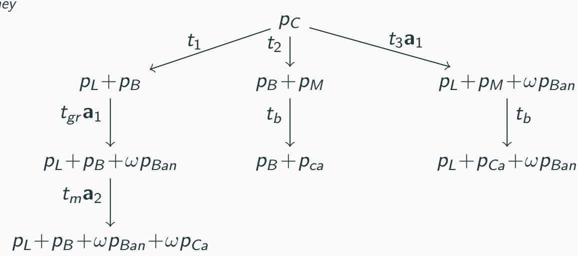

K&M With Accelerations

K&M algorithm:

Variables: (V, E, λ)- a labeled tree; $Front \subseteq V$;

Main loop:

While $Front \neq \emptyset$

Pop $v \in Front$

if $\lambda(v) \leq \lambda(u)$ for $u \in Anc(v)$, **then** Continue

if $\lambda(v) > \lambda(u)$ for $u \in Anc(v)$, **then** Accelerate(u, v)

Explore(v)

K&M With Accelerations

K&M with accelerations algorithm:

Variables: (V, E, λ)- a labeled tree; $Front \subseteq V$;

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Use Acc on $\lambda(v)$

if $\lambda(v) \leq \lambda(u)$ for $u \in Anc(v)$, **then** Continue

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$\mathbf{a} = \text{Accelerate}(u \rightarrow v)$

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$\mathbf{a} =$ Accelerate$(u \to v)$
Acc $=$ Acc $\cup \{\mathbf{a}\}$
Explore(v)

Correctness proof

Similar to our K&M proof!

Can we do better?

Can we do better?

Maximal number of nodes:

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- *Reachability enumeration*: ∞

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Size of Clover is 4

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Can we do better?

Minimal Coverability Tree

Minimal Coverability Tree

MCT[Finkel 93]:

Minimal Coverability Tree

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Improved on K&M algorithm, by keeping V an antichain at any step of the algorithm.

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Minimal Coverability Tree

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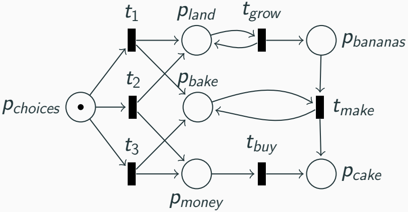
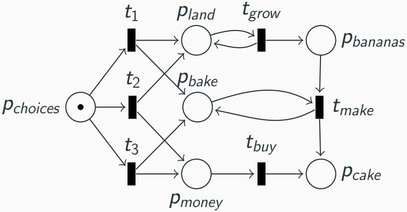
`CovProc`[Geeraerts 10] Alternative construction

Minimal Coverability Tree - MinCov



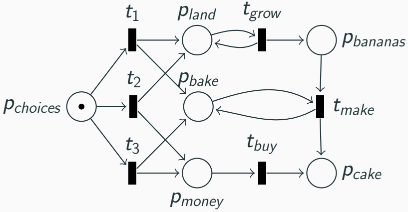
p_C

Minimal Coverability Tree - MinCov



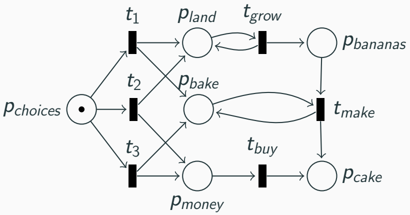
$$p_C \xrightarrow{t_2} p_B + p_M$$

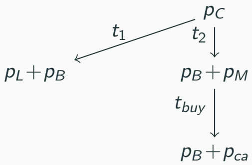
Minimal Coverability Tree - MinCov



$$
\begin{array}{c}
p_C \\
\downarrow t_2 \\
p_B + p_M \\
\downarrow t_{buy} \\
p_B + p_{ca}
\end{array}
$$

Minimal Coverability Tree - MinCov





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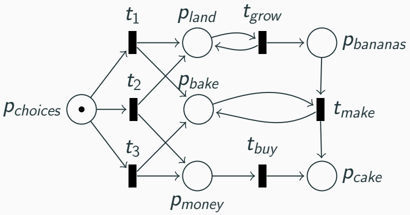

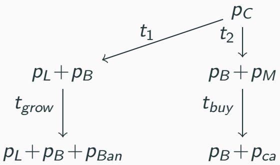

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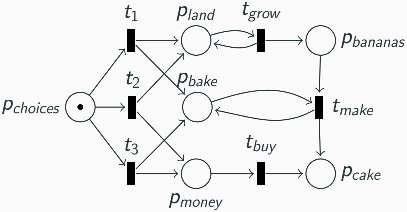

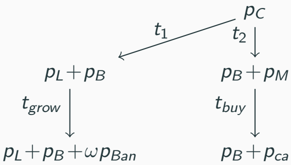

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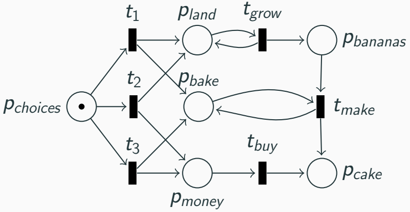

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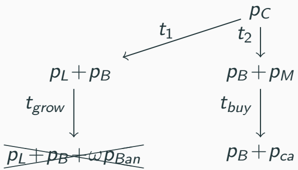

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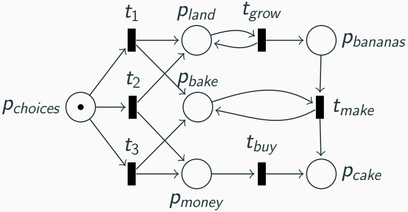

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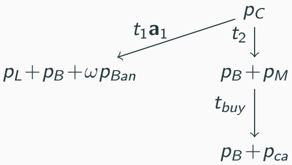

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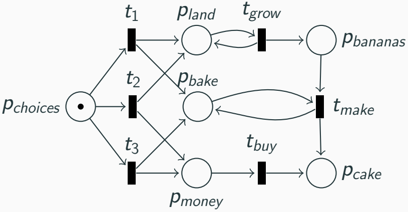

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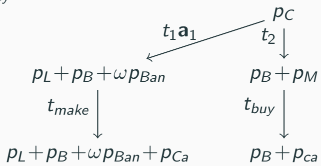

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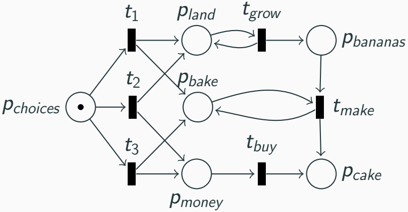

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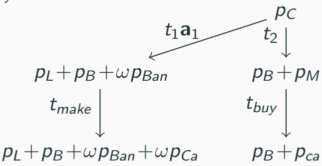

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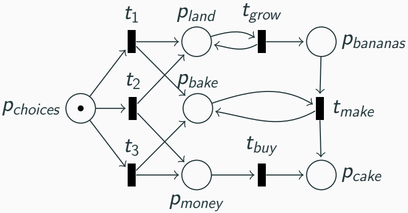

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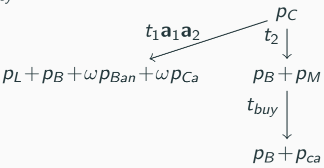

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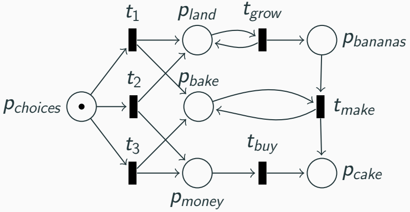

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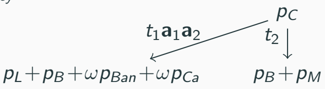

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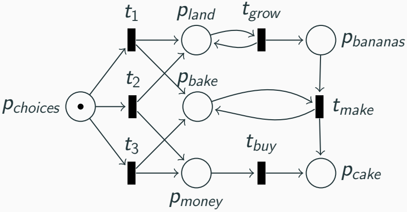

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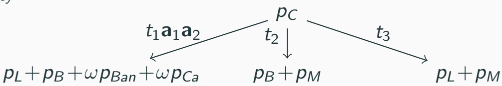

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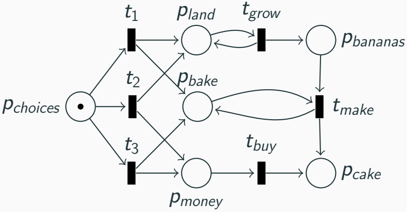

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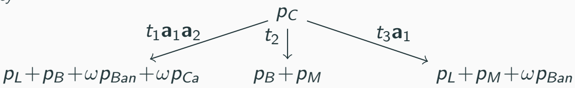

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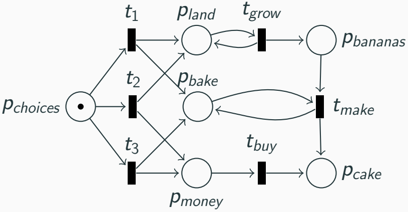

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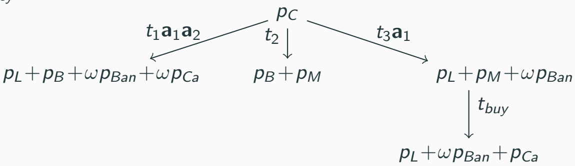

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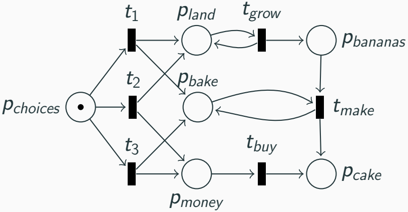

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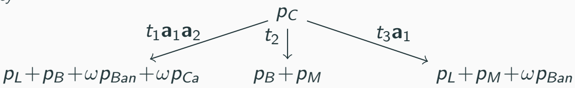

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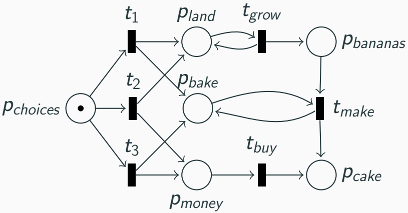

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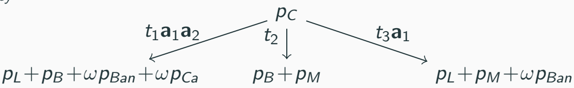

We have the Clover!

K&M With Accelerations

K&M with accelerations algorithm:

Variables: (V, E, λ)- a labeled tree; $Front \subseteq V$;

Acc- a set of ω-transitions;

Main loop:

While $Front \neq \emptyset$

Pop $v \in Front$

Try use Acc on $\lambda(v)$

if $\lambda(v) \leq \lambda(u)$ for $u \in Anc(v)$, **then** Continue

if $\lambda(v) > \lambda(u)$ for $u \in Anc(v)$, **then**

$\mathbf{a} = \text{Accelerate}(u \rightarrow v)$

$\text{Acc} = \text{Acc} \cup \{\mathbf{a}\}$

Explore(v)

K&M With Accelerations

MinCov

Variables: (V, E, λ)- a labeled tree; $Front \subseteq V$;
Acc- a set of ω-transitions;

Main loop:

While $Front \neq \emptyset$
Pop $v \in Front$
Try use Acc on $\lambda(v)$
if $\lambda(v) \leq \lambda(u)$ for $u \in V$, **then** $delete(v)$; Continue
if $\lambda(v) > \lambda(u)$ for $u \in Anc(v)$, **then**
$\mathbf{a}$ = Accelerate($u \rightarrow v$)
Acc = Acc $\cup \{\mathbf{a}\}$
$prune(u)$; Continue
for any $u \in V$, if $\lambda(v) \geq \lambda(u)$ **then** $prune(u)$, $delete(u)$;
Explore(v)

K&M With Accelerations

MinCov

Variables: (V, E, λ)- a labeled tree; $Front \subseteq V$;
Acc- a set of ω-transitions;

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Explore(v)

Correctness proof

K&M With Accelerations

`MinCov`

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Acc- a set of ω-transitions;

Main loop:

While $Front \neq \emptyset$
Pop $v \in Front$
Try use Acc on $\lambda(v)$
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Explore(v)

Correctness proof

Similar to our K&M proof! (with minor modifications)

MinCov

Goals:

Goals:

- Computing the coverability set

MinCov

Goals:

- Computing the coverability set
- Solving the coverability problem

MinCov

Goals:

- Computing the coverability set
- Solving the coverability problem

Implementation features

Goals:

- Computing the coverability set
- Solving the coverability problem

Implementation features

- Written in Python3, using the Numpy and Z3-solver libraries.
- $\approx$ 2000 lines.
- Imports Petri nets in ".spec" format from Mist.
- Can be found in https://github.com/IgorKhm/MinCov

Benchmarks

123 benchmarks (literature)

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1078 benchmarks (random)

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- MinCov only 1.2 slower then the fastest tool.

Comparing MinCov and qCover for Coverability

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Blondin et al. (`qCover`) (2016)

Combining backward exploration with forward over-approximation

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Complementary tools!

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MinCov ∥ qCover[1]	1841	2	13493	11	13	15334

1. $\text{Time}(\texttt{MinCov} \parallel \texttt{qCover}) = 2\min(\text{Time}(\texttt{MinCov}), \text{Time}(\texttt{qCover}))$.

Conclusion and Future Work

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- $\exists A \subset \text{Acc}$ such that $\uparrow A = \text{Acc}$ and $|A| \leq 3{-}EXP(\mathcal{N})$