Coverability and Termination in Recursive Petri Nets

Petri Nets 2019

Alain Finkel, Serge Haddad, Igor Khmelnitsky

Petri nets

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Hardness of modeling:

- Exceptions
- Faults
- Interrupts

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Expressiveness: $CFL \nsubseteq PNL \nsubseteq CFL$.

Recursive Petri nets

Introduced in [EH96] (around 100 citations).

Modeling:

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Modeling:

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Decidability:

- Reachability
- Boundedness
- Termination

Expressiveness: $CFL \subsetneq RPNL$.

Related work

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	PN+Stack	BVASS

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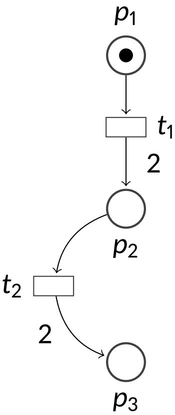
	PN+Stack	BVASS
Reachability	TOWER-hard	TOWER-hard
Coverability	TOWER-hard	2-EXPTIME-complete
Boundedness	Decidable	2-EXPTIME-complete
Termination	Decidable	?

Outline

From Petri Net to Recursive Petri Net

A Petri net



A marking

p_1

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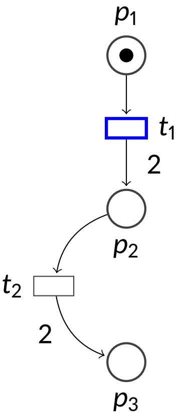

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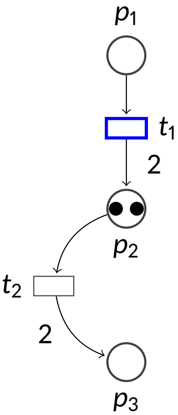

A marking

$2p_2$

From Petri Net to Recursive Petri Net

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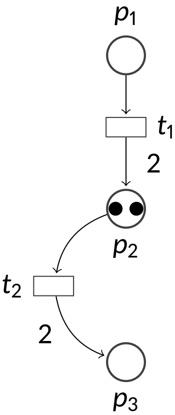

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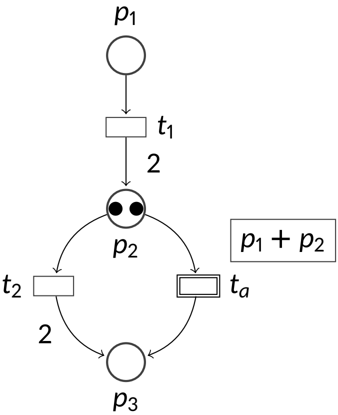

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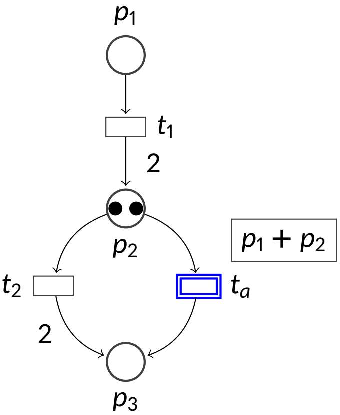

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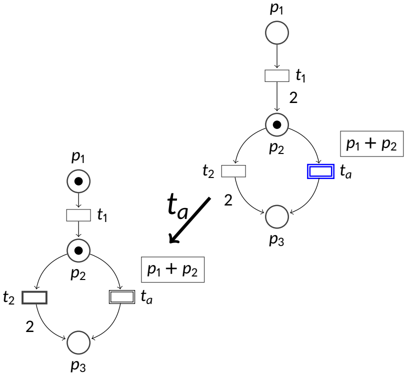

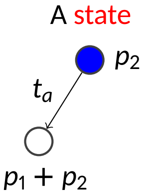

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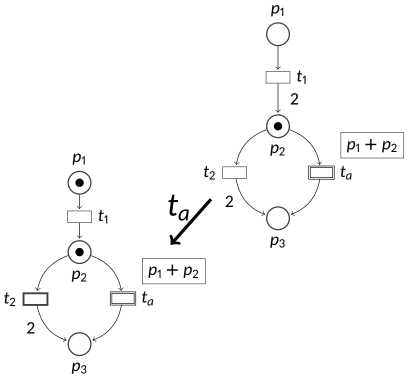

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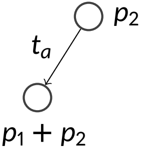

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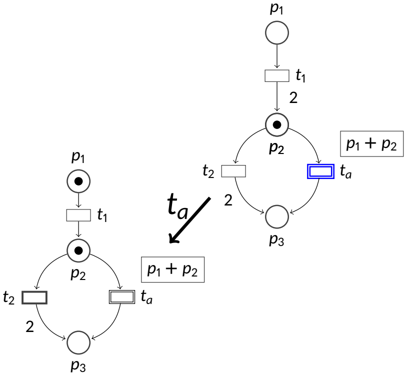

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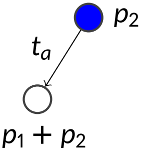

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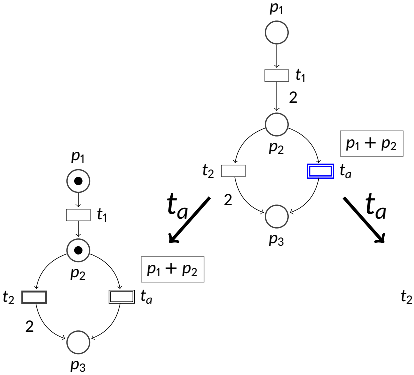

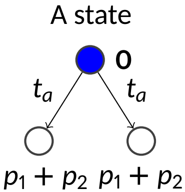

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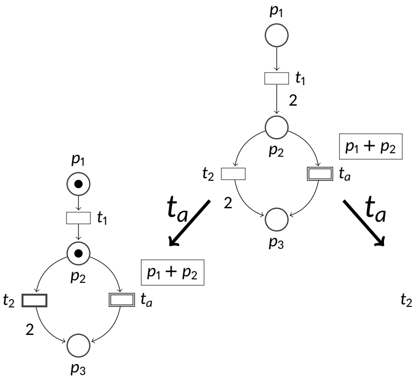

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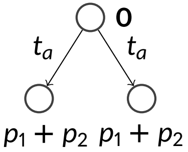

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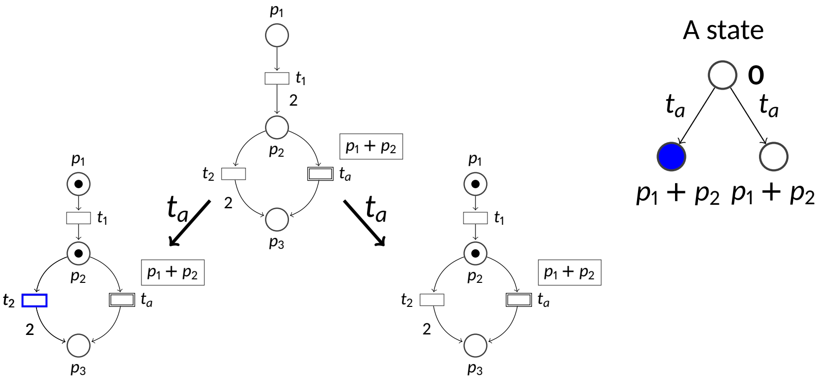

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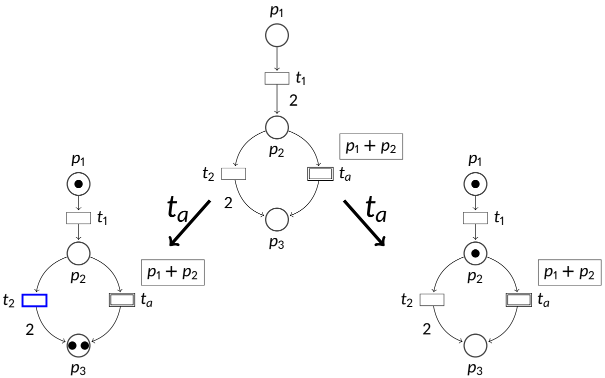

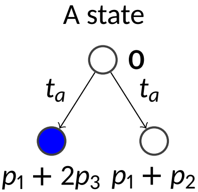

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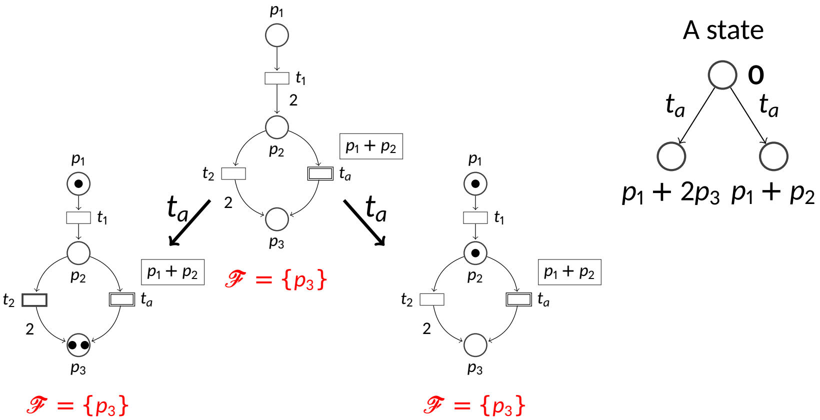

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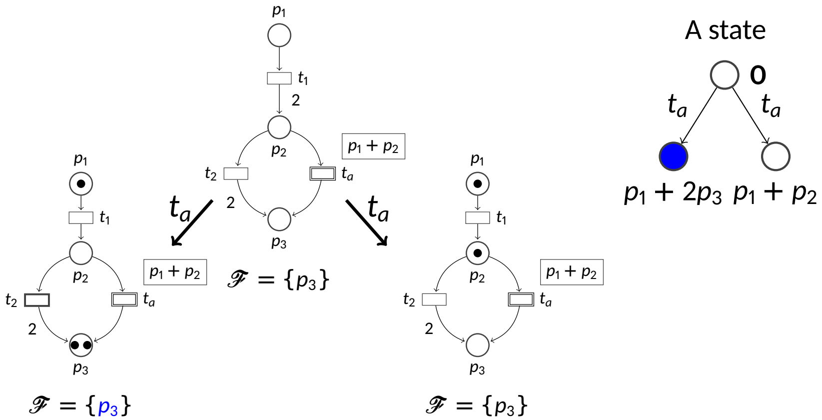

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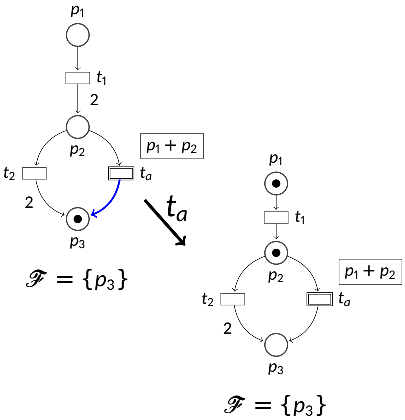

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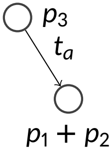

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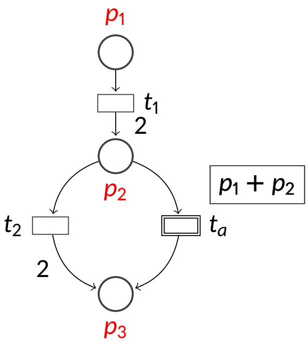
- $P = \{p_1, p_2, p_3\}$
- $T = T_{el} \uplus T_{ab} = \{t_1, t_2\} \uplus \{t_a\}$
- W^- and W^+ incidence matrices:
  $W^-(t_1) = p_1, W^+(t_1) = 2p_2$
- $\Omega : T_{ab} \rightarrow \mathbb{N}^P$ starting markings:
  $\Omega(t_a) = p_1 + p_2$
- $\mathscr{F}$ final markings:
  $\mathscr{F} = \{p_3\}$



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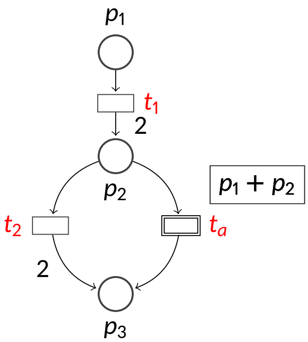

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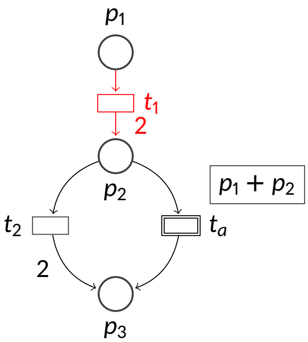

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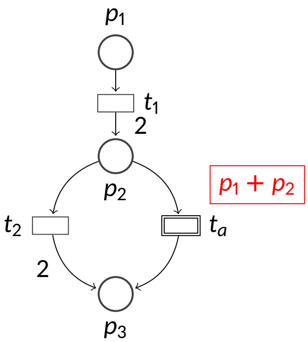

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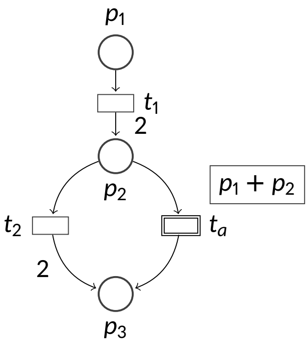

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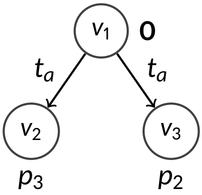

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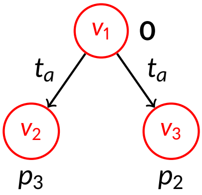

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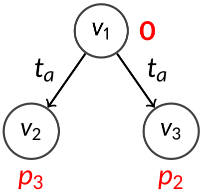

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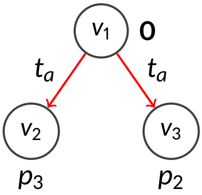

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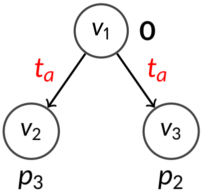
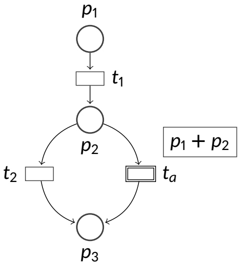


Firing in an RPN

Three types of transitions:



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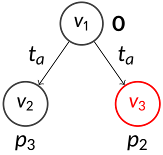

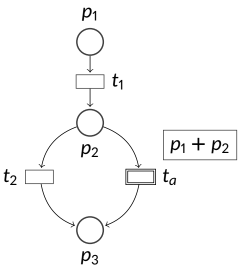

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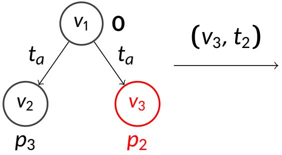

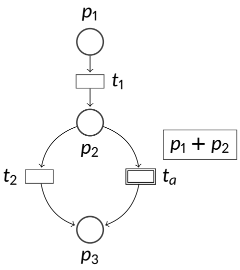

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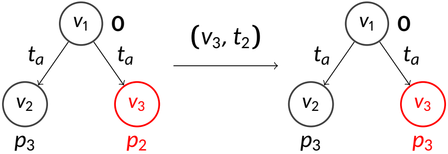

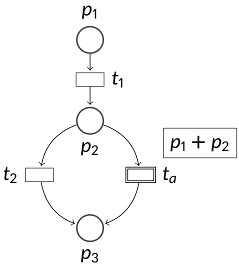

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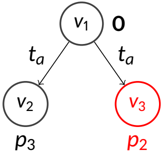

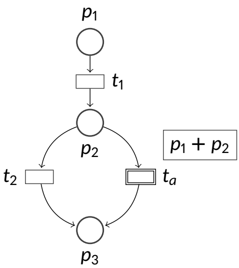

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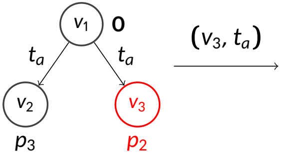

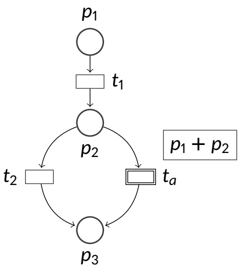

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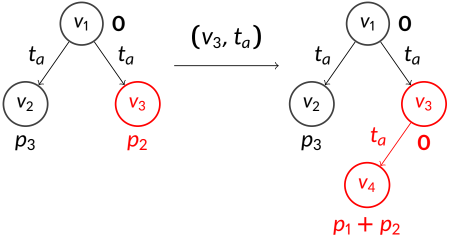

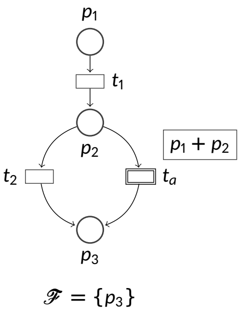

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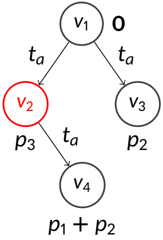

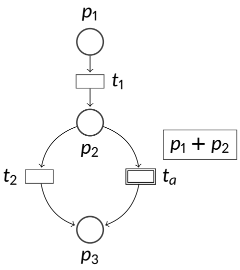

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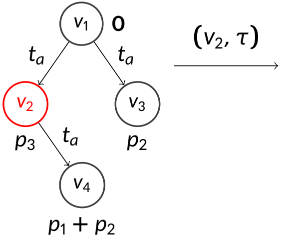

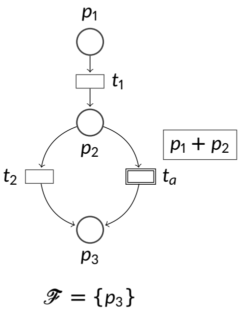

Firing in an RPN

Three types of transitions:

1. Elementary transition
2. Abstract transition
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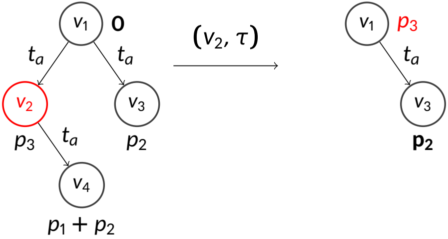

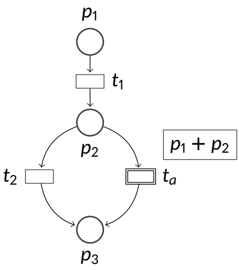

$\mathscr{F} = \{p_3\}$

Firing in an RPN

Three types of transitions:

1. Elementary transition
2. Abstract transition
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A firing sequence:

$$s_0 \xrightarrow{(v_1,t_1)} s_1 \xrightarrow{(v_2,t_2)} \cdots s_{n-1} \xrightarrow{(v_n,t_n)} s_n$$

Or equivalently $s_0 \xrightarrow{\sigma} s_n$ for $\sigma = (v_1, t_1)(v_2, t_2)\ldots(v_n, t_n)$.

Goals of the paper

Goals of the paper

Order

Goals of the paper

Order How do we order states?

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Expressiveness

Goals of the paper

Order How do we order states?

Expressiveness How expressive are RPN coverability languages?

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Complexity 1. Coverability problem?

Goals of the paper

Order	How do we order states?
Expressiveness	How expressive are RPN coverability languages?
Complexity	1. Coverability problem?
	2. Termination problem?

Outline

Ordering states

$s_i = \langle V_i, M_i, E_i, \Lambda_i \rangle$

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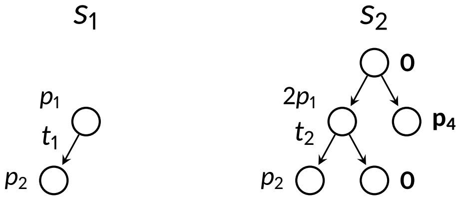

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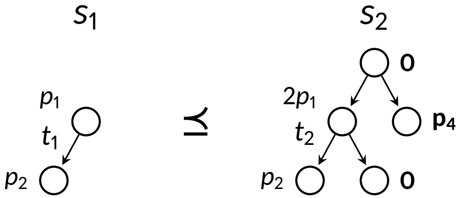

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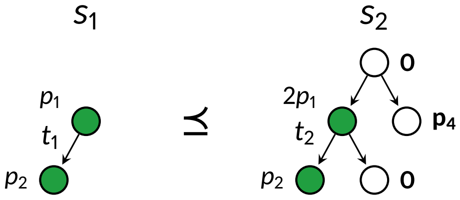

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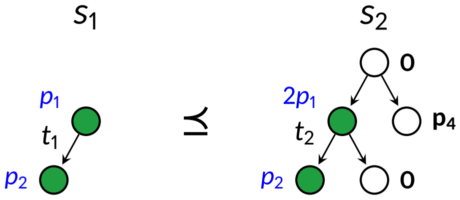

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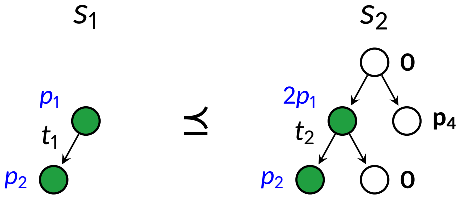

$$W^+(t_1) = p_1 \leq 2p_1 = W^+(t_2)$$

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2. For every $v \in V_1$, $M_1(v) \leq M_2(f(v))$.
3. For every $e \in E_1$, $W^+(\Lambda_1(e)) \leq W^+(\Lambda_2(f(e)))$.

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\rotatebox{90}{$\preceq$} & & \\
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\end{array}
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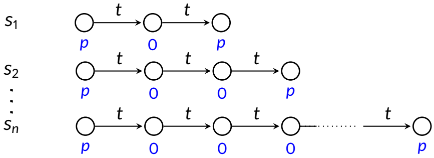

RPN coverability languages

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- $\mathcal{N}$ - RPN
- s_0 - initial state
- S_f - finite set of states
- $\lambda : T^* \rightarrow \Sigma^*$ - morphism function.

Comparison

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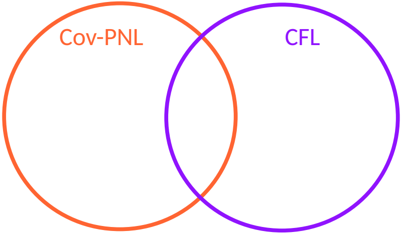

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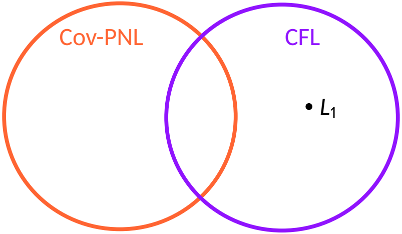

$$L_1 = \{w \in \{d, e\}^* \mid w = \widetilde{w}\}$$

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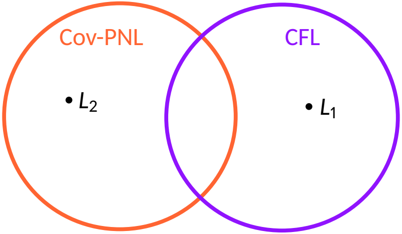

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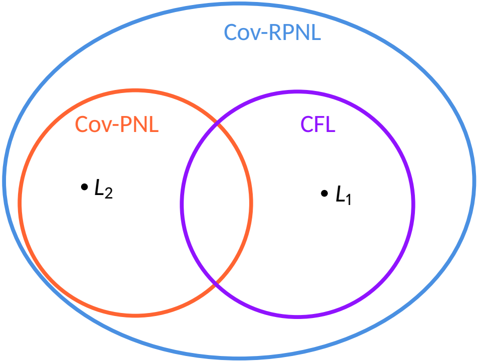

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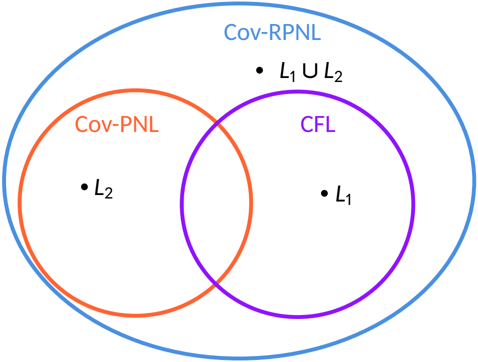

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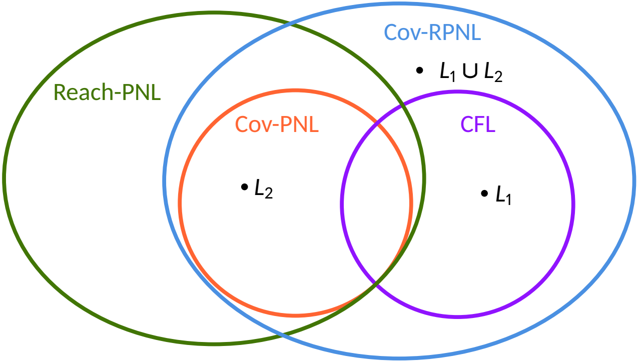

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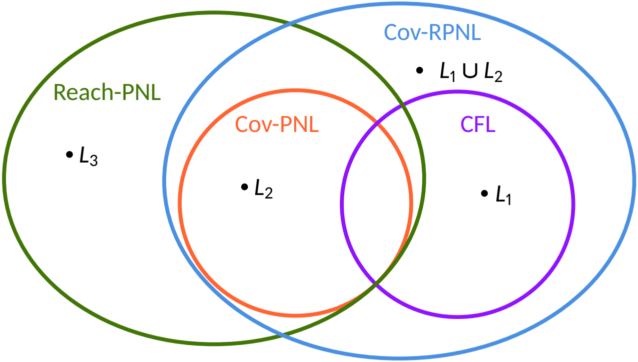

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Outline

Coverability problem - RPN

Given an *RPN* and s_0, s_f two states.

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From this proposition and Savitch's theorem, the coverability problem of *RPN* is EXPSPACE-complete.

Coverability problem - PN

Given an *PN* and m_0, m_f two markings.

$$\exists \sigma \ m_0 \xrightarrow{\sigma} m \succeq m_f \,?$$

Proposition:[Rac78]

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$$\sigma' = \sigma_1(v_1, \tau)\sigma_2(v_2, \tau) \ldots \sigma_\ell(v_\ell, \tau)\sigma_{\ell+1}\sigma_{\ell+1}^{ab} \ldots \sigma_k\sigma_k^{ab}$$

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Applying Rackoff's proposition to each σ_i,

we get $s_0 \xrightarrow{\sigma''} s'' \succeq s_f$, s.t. $|\sigma''| \leq 2^{2^{cn\log n}}$.

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  Reduced to small number of PN termination problems.

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- Complexity: Coverability and termination problems are EXPSPACE-complete.

Future works

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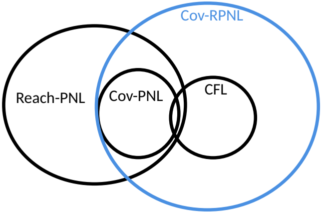

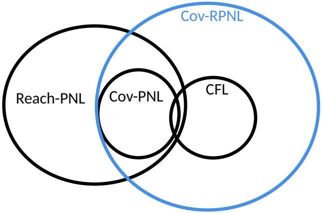

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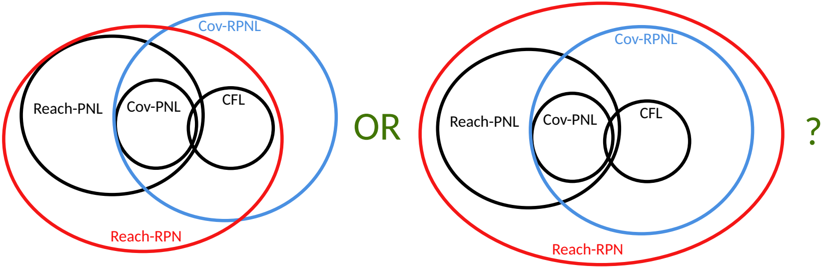

Future works

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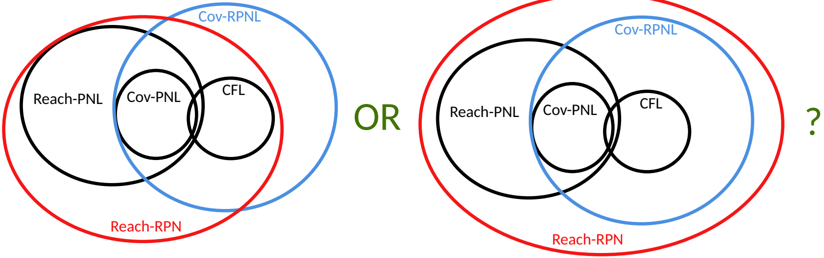

Questions?

	PN+Stack	BVASS	RPN
Reachability	TOWER-hard	TOWER-hard	Decidable
Coverability	TOWER-hard	2-EXPTIME-complete	EXPSPACE-complete
Boundedness	Decidable	2-EXPTIME-complete	?
Termination	Decidable	?	EXPSPACE-complete

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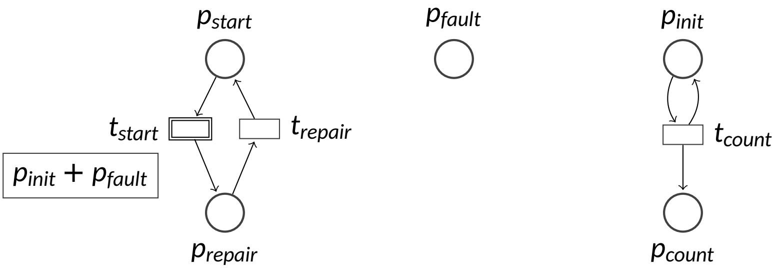
[Rac78] Charles Rackoff, *The covering and boundedness problems for vector addition systems*, Theoretical Computer Science **6** (1978), no. 2, 223 – 231.

Fault tolerant system



$$\mathscr{F} = \{p_{fault}\}$$